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AN INVERSE LAPLACE TRANSFORMATION FOR SOLVING HEAT-CONDUCTION PROBLEMS WITH DISCONTINUOUS BOUNDARY CONDITIONS OF THE SECOND KIND

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An inverse Laplace transformation is found for a class of functions encountered in heat-conduction problems with discontinuous boundary conditions.

In the solution of multidimensional axisymmetric nonstationary heat-conduction problems [1-3] for a system of two semibounded bodies with different thermophysical characteristics (TPC) in thermal contact in a plane wherein bounded (local) surface heat sources are operative with arbitrarily specified laws of heat flow density measurement in the corresponding domains, Laplace transform representations of the following form are encountered:

$$\mathrm{Lv}(s)=\mathrm{Lv}\left(s\left|\begin{matrix}p_1, & p_2, & p_3, & \nu\\ k_1, & k_2, & k_3 & \end{matrix}\right.\right)=\frac{1}{s^{\nu}}\frac{1-k_1\exp(-p_1\sqrt{s})}{1-k_2\exp(-p_2\sqrt{s})-k_3\exp(-p_3\sqrt{s})}, \tag{1}$$
$$\nu,\ p_i>0,\ i=1,\ 2,\ 3.$$

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In the present paper we examine the problem of inverting the transform (1). The resulting original function is represented in the form of a nonsingular integral expressible in terms of a dual series in degenerate hypergeometric functions [4].

LEMMA. Let $p_i > 0$, $i = \overline{1, 3}$, and $k_2 k_3 \geq 0$. Then for $|k_2 + k_3| < 1$, or for $|k_2 + k_3| = 1$, but with $k_1 = 1$, the modulus of the function

$$\Phi(s) = \frac{1 - k_1 \exp(-p_1\sqrt{s})}{1 - k_2 \exp(-p_2\sqrt{s}) - k_3 \exp(-p_3\sqrt{s})} \tag{2}$$

is bounded in the angle $S = \{s: |\arg s| < \pi\}$.

Proof. We determine the conditions under which the denominator of the function (2) does not vanish in the angle S. It is obvious that equating the denominator to zero is equivalent to the system

$$k_2 \exp(-p_2 \mathrm{Re}\sqrt{s}) \cos(p_2 \mathrm{Im}\sqrt{s}) + k_3 \exp(-p_3 \mathrm{Re}\sqrt{s}) \cos(p_3 \mathrm{Im}\sqrt{s}) = 1,$$
$$k_2 \exp(-p_2 \mathrm{Re}\sqrt{s}) \sin(p_2 \mathrm{Im}\sqrt{s}) + k_3 \exp(-p_3 \mathrm{Re}\sqrt{s}) \sin(p_3 \mathrm{Im}\sqrt{s}) = 0,$$

whence we have

$$k_2^2 \exp(-2p_2 \mathrm{Re}\sqrt{s}) + k_3^2 \exp(-2p_3 \mathrm{Re}\sqrt{s}) + 2k_2 k_3 \times$$
$$\times \exp[-(p_2 + p_3) \mathrm{Re}\sqrt{s}] \cos[(p_2 - p_3) \mathrm{Im}\sqrt{s}] = 1.$$

We estimate the left side of the last equation. It is not difficult to see that the value of the left side of this equation satisfies the condition $\leq (k_2 + k_3)^2$, the equality sign being attained only for $s = 0$. Consequently, if $|k_2 + k_3| < 1$, the denominator of the function (2) has no zeros in the angle S. We show that even when $|k_2 + k_3| = 1$, there are no zeros. For this, it is obviously sufficient to prove the existence of the limit of $\Phi(s)$ as $s \to 0$.

We have

$$\lim_{\substack{s\to 0\\ s\in S}} \Phi(s) = \lim_{\substack{s\to 0\\ s\in S}} \frac{1 - k_1 + k_1 p_1 \sqrt{s} + O(s)}{1 - k_2 - k_3 + (k_2 p_2 + k_3 p_3)\sqrt{s} + O(s)} =$$
$$= \begin{cases} \dfrac{1 - k_1}{1 - k_2 - k_3}, & \text{if} \quad k_2 + k_3 \neq 1, \\ \dfrac{k_1 p_1}{k_2 p_2 + k_3 p_3}, & \text{if} \quad k_2 + k_3 = k_1 = 1. \end{cases}$$

It remains to prove the boundedness of the modulus of $\Phi(s)$ as $s \to \infty$ along the boundary of the angle S, i.e., $\sqrt{s} = R\exp[i(\pi/2 - \varepsilon)]$, $\varepsilon > 0$:

$$\lim_{\substack{s\to\infty\\ s\in\partial S}} \Phi(s) = \lim_{\substack{R\to\infty\\ \varepsilon>0}} \frac{1 - \exp\{-p_1 R \exp[i(\pi/2 - \varepsilon)]\}}{1 - k_2 \exp\{-p_2 R \exp[i(\pi/2 - \varepsilon)]\} - k_3 \exp\left\{-p_3 R \exp\left[i\left(\frac{\pi}{2} - \varepsilon\right)\right]\right\}} = 1.$$

In accordance with the Phragmen–Lindelöf principle, it follows that $\Phi(s)$ is bounded in the angle S. Thus the lemma is proved.

We now go to the immediate inversion of the function (1):

$$\mathrm{Lv}(\tau) = L^{-1}[\mathrm{Lv}(s)] = \lim_{\delta\to\infty} \frac{1}{2\pi i} \int_{\sigma - i\delta}^{\sigma + i\delta} \exp(s\tau)\, \mathrm{Lv}(s)\, ds, \quad \sigma > 0.$$

Under the conditions stated in the lemma, the integrand function is single-valued and analytic in the angle S. Consequently, by the Cauchy theorem (Fig. 1),

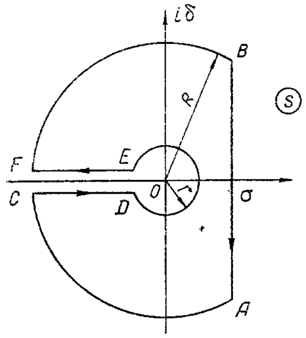

Fig. 1

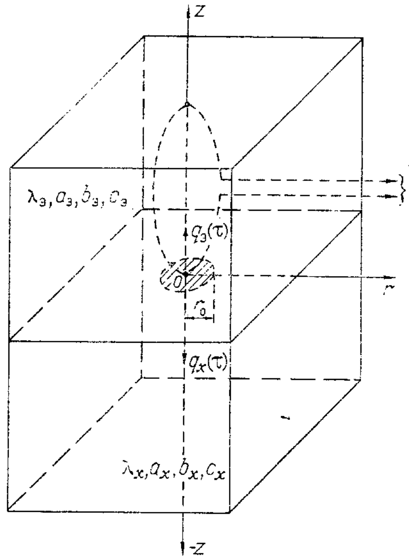

Fig. 2

Fig. 1. Contour for calculating the inverse Laplace transform of the function (1).

Fig. 2. Idealized physical model for the contact of semibounded bodies with a thin circular heat source.

$$\int\limits_{\sigma-i\delta}^{\sigma+i\delta} = \int\limits_{AC} + \int\limits_{FB} + \int\limits_{CD} + \int\limits_{EF} + \int\limits_{DE} .$$

A detailed evaluation of these integrals is required; an evaluation of the second has already been given in Sec. 11.2 of [5]. In view of this, we give only the main features in the inversion of the function (1).

It follows from the above lemma and the Jordan lemma that the integrals along the large arcs AC and FB tend toward zero. The integral along the small circle DE tends toward a constant:

$$A = \begin{cases} \dfrac{1-k_1}{1-k_2-k_3}, & \text{if} \quad k_2+k_3 \neq 1, \ \nu = 1, \\ \dfrac{p_1}{k_2p_2+k_3p_3}, & \text{if} \quad k_2+k_3 = k_1 = 1, \ \nu = 1, \\ 0, & \text{if} \quad 0 < \nu < 1. \end{cases}$$

In evaluating the integral along the segments CD and EF, it is necessary to take into account that along the lower edge of the cut the function s^ν takes on the value $|s|^\nu \exp(-\pi\nu i)$, while on the upper edge it takes on the value $|s|^\nu \exp(\pi\nu i)$. Thus, we can represent the original function (1) in the form

$$\begin{gathered} \mathrm{Lv}(\tau) = \mathrm{Lv}\left(\tau \left| \begin{matrix} p_1, & p_2, & p_3, & \nu \\ k_1, & k_2, & k_3 & \end{matrix} \right.\right) = A + \frac{2}{\pi} \int\limits_0^\infty \frac{\exp(-\tau x^2)}{x^{2\nu-1}} \times \\ + [\sin \pi\nu + k_2 \sin(p_2x - \pi\nu) + k_3 \sin(p_3x - \pi\nu) - k_1 \sin(p_1x + \pi\nu) + \\ + k_1k_2 \sin(p_1x - p_2x + \pi\nu) + k_1k_3 \sin(p_1x - p_3x + \pi\nu)]/[1 + \\ + k_2^2 + k_3^2 - 2k_2 \cos p_2x - 2k_3 \cos p_3x + 2k_2k_3 \cos(p_2x - p_3x)]\, dx. \end{gathered} \tag{3}$$

We represent the function $\mathrm{Lv}(\tau)$ in terms of a dual series in degenerate hypergeometric functions. To do this we split up the integral on the right side of Eq. (3) into two integrals, gathering separately the terms for $\sin \pi\nu$ and $\cos \pi\nu$, and we then differentiate the left and right sides of Eq. (3) with respect to τ. Justification for the differentiation of the integral with respect to the parameter τ follows from the fact that the integrals obtained thereby converge uniformly with respect to τ on an arbitrary finite interval for $0 < \nu \leq 1$:

$$\frac{\partial}{\partial\tau}\mathrm{Lv}(\tau)=-\frac{\sin\pi\nu}{\pi}\int_0^\infty x^{3-2\nu}\exp(-\tau x^2)\left[\frac{1-k_1\exp(-ip_1x)}{1-k_2\exp(-ip_2x)-k_3\exp(-ip_3x)}+\right.$$
$$\left.+\frac{1-k_1\exp(ip_1x)}{1-k_2\exp(ip_2x)-k_3\exp(ip_3x)}\right]dx-$$
$$-\frac{\cos\pi\nu}{\pi}\int_0^\infty x^{3-2\nu}\exp(-\tau x^2)\left[\frac{1-k_1\exp(ip_1x)}{1-k_2\exp(ip_2x)-k_2\exp(ip_3x)}-\right.$$
$$\left.-\frac{1-k_1\exp(-ip_1x)}{1-k_2\exp(-ip_2x)-k_3\exp(-ip_3x)}\right]dx.$$

We now expand the function $[1-k_2\exp(\pm ip_2x)-k_3\exp(\pm ip_3x)]^{-1}$, subject to the condition $1>|k_2\exp(\pm ip_2x)+k_3\exp(\pm ip_3x)|=\sqrt{k_2{}^2+k_3{}^2+2k_2k_3\cos(p_2x-p_3x)}\geqq|k_2-k_3|$, in a dual series with respect to x on an arbitrary finite interval for $0<\nu\leqq 1$, we interchange the order of integration and summation. We note here that it is not possible to expand the integrand function on the right side of Eq. (3) since the resulting dual series, together with $\exp(-\tau x^2)x^{1-2\nu}$ has a point of nonuniform convergence at $x=0$. We have

$$\frac{\partial}{\partial\tau}\mathrm{Lv}(\tau)=-\frac{2}{\pi}\sum_{n=0}^\infty\sum_{m=0}^n k_2^m k_3^{n-m}\binom{n}{m}\Big\{\sin\pi\nu\int_0^\infty x^{3-2\nu}\exp(-\tau x^2)\times$$
$$\times[\cos xa_{mn}-k_1\cos xa_{mn}]\,dx+\cos\pi\nu\int_0^\infty x^{3-2\nu}\exp(-\tau x^2)\ [\sin xa_{mn}-k_1\sin xa_{mn}]\,dx\Big\},$$

where $a_{mn}=p_2m+p_3(n-m)$.

We integrate the left and right sides of the latter equation with respect to τ and then evaluate the resulting integrals:

$$\mathrm{Lv}(\tau)=\mathrm{Lv}\left(\tau\,\middle|\,\begin{matrix}p_1,\ p_2,\ p_3,\ \nu\\ k_1,\ k_2,\ k_3\end{matrix}\right)=\sum_{n=0}^\infty\sum_{m=0}^n k_2^m k_3^{n-m}\binom{n}{m}\left\{\frac{\mathrm{sgn}(1-\nu)}{\Gamma(\nu)\tau^{1-\nu}}\left[{}_1F_1\left(1-\nu;\ \frac{1}{2};\ -\frac{a_{mn}^2}{4\tau}\right)-\right.\right.\tag{4}$$
$$\left.-k_1\,{}_1F_1\left(1-\nu;\ \frac{1}{2};\ -\frac{a_{mn}^2}{4\tau}\right)\right]+\frac{1}{\Gamma\left(\nu-\frac{1}{2}\right)\tau^{3/2-\nu}}\left[k_1a_{mn}\,{}_1F_1\left(\frac{3}{2}-\nu;\ \frac{3}{2};\ -\frac{a_{mn}^2}{4\tau}\right)-\right.$$
$$\left.\left.-a_{mn}\,{}_1F_1\left(\frac{3}{2}-\nu;\ \frac{3}{2};\ -\frac{a_{mn}^2}{4\tau}\right)\right]\right\},$$

where ${}_1F_1(a;\ b;\ x)$ is a degenerate hypergeometric function [4]; $a_{mn}=p_2m+p_3(n-m)$,

$$\mathrm{sgn}(1-\nu)=\begin{cases}1, & 0<\nu<1,\\ 0, & \nu=1.\end{cases}$$

As is evident, all the operations made above are valid for $0<\nu\leqq 1$. However, the inverse Laplace transform of the function (1) exists for arbitrary $\nu>0$. Therefore, to invert the transform (1) for $\nu>1$, it is necessary to use the convolution theorem [5, 4]. Thus, we have the following theorem.

THEOREM. The inverse Laplace transform of function (1) is the function $\mathrm{Lv}\left(\tau\,\middle|\,\begin{matrix}p_1,\ p_2,\ p_3,\ \nu\\ k_1,\ k_2,\ k_3\end{matrix}\right)$, where $p_i>0$, $i=\overline{1,3}$, $0<\nu\leqq 1$, which may be expressed in the form of the integral (3) if $k_2k_3\geqq 0$ and $|k_2+k_3|<1$, or if $|k_2+k_3|=1$, but with $k_1=1$, or in the form of the dual series (4) subject to the supplementary condition $|k_2-k_3|<1$.

We give several examples of the use of the inverse Laplace transform of the function (1) for solving a specific heat-conduction problem.

Assume that we have two half-spaces (Fig. 2) with different TPC, which are in ideal thermal contact with a thin circular heat source of constant power $q_0(\tau)=q_0=W_0/\pi r_0{}^2$, where r_0 is the radius of action of the source. The initial temperature distribution at all points of the system of bodies considered is uniform and equal to $T_0=\mathrm{const}$. Outside the circle $(r>r_0)$ in the plane $(z=0)$ of contact of the given bodies the temperature gradient along the normal to the boundary separating the bodies is absent. We are required to determine: 1) the dependence of the nonstationary specific thermal flows $q_x(\tau)$ or $q_e(\tau)$ on the axis $r = 0$, advancing into the corresponding half-space due to the heating of the given system of bodies by a bounded heat source of constant power; 2) the dependence of the nonstationary temperature at the central point ($r = z = 0$) of the circular heat source.

The value of the transform $\bar{q}_x(s)$ for the specific heat flow, directed along the $|z|$ axis ($r = 0$) and advancing into the semibounded domain of the body in question, may be expressed, due to the action, in the system of bodies considered, of the circular heat source of arbitrarily specified specific power $q_0(\tau) = L^{-1}[\bar{q}_0(s)]$, in the following form:

$$\bar{q}_x(s) = \bar{q}_0(s)\,\frac{1-\exp(-\beta_э\sqrt{s})}{1+k_b^{-1}-\exp(-\beta_e\sqrt{s})-k_b^{-1}\exp(-\alpha_x\sqrt{s})}, \tag{5}$$

where $\bar{q}_x(s)=\int\limits_0^\infty q_x(\tau)\exp(-s\tau)\,d\tau$; $\bar{q}_0(s)=\int\limits_0^\infty q_0(\tau)\exp(-s\tau)\,d\tau$ is the transform of the total specific heat flow $q_0(\tau)$, generated by the given heat source of arbitrarily specified (in time) power; $\beta_e = r_0/\sqrt{a_e}$, $\alpha_x = r_0/\sqrt{a_x}$, $k_b^{-1} = b_e/b_x$, r_0 is the radius of the thin circular heat source; a_x, b_x, a_e, b_e are, respectively, the coefficients of thermal diffusivity and thermal activity of the bodies considered (subscript x refers to the body in question and subscript e refers to a standard body).

For the case in which $q_0(\tau) = q_0 = \text{const}$, the transform $L[q_0] = q_0/s$, and expression (5), in accordance with Eq. (1), may be written as follows:

$$\frac{\bar{q}_x(s)}{q_0} = \frac{1}{1+k_b^{-1}}\,\mathrm{Lv}\left(s\left|\begin{matrix}\beta_e, & \alpha_x, & \beta_e, & 1\\ 1, & \dfrac{1}{1+k_b}, & & \dfrac{k_b}{1+k_b}\end{matrix}\right.\right). \tag{6}$$

With the help of the known inverse Laplace transform of function (1) we can readily write formula (6) in the form

$$\frac{q_x(\tau)}{q_0} = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\binom{n}{m}k_b^{n-m+1}}{(1+k_b)^{n+1}}\left[\operatorname{erfc}\frac{nk_a^{1/2}+m(1-k_a^{1/2})}{2\sqrt{\mathrm{Fo}_x}}-\operatorname{erfc}\frac{(n+1)k_a^{1/2}+m(1-k_a^{1/2})}{2\sqrt{\mathrm{Fo}_x}}\right], \tag{7}$$

since ${}_1F_1\left(\frac{1}{2};\ \frac{3}{2};\ -x^2\right)=\frac{\sqrt{\pi}}{2x}\operatorname{erf}x$, $k_a = a_x/a_e$.

Since for the thermal flows $q_0(\tau)$, $q_x(\tau)$, $q_e(\tau)$ we always have the relation $q_0(\tau) = q_x(\tau) + q_e(\tau)$, we find, using Eq. (7), an expression also for the specific heat flow $q_e(\tau)$.

The transform $\bar{T}(0,\ 0,\ s) - (T_0/s) = \Delta\bar{T}(s)$ for the excess temperature at the center ($r = z = 0$) of the heating spot for the set of given bodies in thermal contact has, even in the case of the arbitrarily timewise-specified specific power $q_0(\tau)$ generated by the circular heat source, the form

$$\Delta\bar{T}(s) = b_x^{-1}\,\frac{\bar{q}_0(s)}{\sqrt{s}}\,\frac{[1-\exp(-\alpha_x\sqrt{s})][1-\exp(-\beta_э\sqrt{s})]}{1+k_b^{-1}-\exp(-\beta_e\sqrt{s})-k_b^{-1}\exp(-\alpha_x\sqrt{s})}. \tag{8}$$

All the notation used in Eq. (8) corresponds to that used in formula (5). For the given heat source of constant specific power $L[q_0] = q_0/s$ the transform (8), in accordance with Eq. (1), may be written as

$$\Delta\bar{T}(s) = \frac{q_0}{b_x}\,\bar{\varphi}(s)\,\mathrm{Lv}\left(s\left|\begin{matrix}\beta_e & \alpha_x, & \beta_e, & 1\\ 1, & \dfrac{1}{1+k_b}, & & \dfrac{k_b}{1+k_b}\end{matrix}\right.\right), \tag{9}$$

where $\bar{\varphi}(s) = (1/\sqrt{s})[1 - \exp(-\alpha_x\sqrt{s})]$.

Using the inverse Laplace transform of the function $\bar{\varphi}(s)$ [6],

$$\varphi(\tau)=L^{-1}[\bar{\varphi}(s)]=\frac{1}{\sqrt{\pi\tau}}[1-\exp(-\alpha_x^2/4\tau)]$$

and formula (4), we obtain the inverse Laplace transform of the transform (9):

$$\Delta T(\tau)=L^{-1}[\Delta\bar{T}(s)]=\frac{q_0}{b_x}\int_0^\tau \varphi(\xi)\,\mathrm{Lv}\left(\tau-\xi\left|\begin{matrix}\beta_e, & \alpha_x, & \beta_e, & 1 \\ 1, & \frac{1}{1+k_b}, & \frac{k_b}{1+k_b} & \end{matrix}\right.\right)d\xi=$$

$$=\frac{2q_0\sqrt{\tau}}{b_e}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\binom{n}{m}k_b^{n-m}}{(1+k_b)^{n+1}}\left[\operatorname{ierfc}\frac{n+m(k_a^{-1/2}-1)}{2\sqrt{\mathrm{Fo_e}}}-\right.$$

$$-\operatorname{ierfc}\frac{n+1+m(k_a^{-1/2}-1)}{2\sqrt{\mathrm{Fo_e}}}-\operatorname{ierfc}\frac{n+k_a^{-1/2}+m(k_a^{-1/2}-1)}{2\sqrt{\mathrm{Fo_e}}}+$$

$$\left.+\operatorname{ierfc}\frac{n+1+k_a^{-1/2}+m(k_a^{-1/2}-1)}{2\sqrt{\mathrm{Fo_e}}}\right].$$

Thus, the inverse Laplace transform of the class of function transforms of the form (1) allows us to reduce the solutions of some analogous nonstationary heat-conduction problems to tabular form.

NOTATION

Lv(s), designation, assumed by the present authors, of the class of function transforms under consideration; Lv(τ), designation, assumed by the present authors of the class of function inverse transforms; p_i, k_i, parameters characterizing the relationship between thermal (physical) properties of the model of a system of bodies under consideration and allowing for characteristic sizes of inner heat sources; ν, parameter characterizing a given change of heat flux density; s, parameter of integral Laplace transformation; τ, time; Re$\sqrt{s}$, real part of $\sqrt{s}$; Im$\sqrt{s}$, imaginary part of $\sqrt{s}$; R, radius-vector (modulus $\sqrt{s}$); ${}_1F_1(a;\ b;\ x)$, designation of the degenerate hypergeometric function (Kummer function); $\varkappa_a = a_x/a_e$; $k_b = b_x/b_e$; $b_x = \lambda_x/\sqrt{a_x}$; $b_e = \lambda_e/\sqrt{a_e}$; thermal conductivities; a_x, a_e, thermal diffusivities; $\bar{q}_c(s)$ (c = x, e, 0), representations of the corresponding heat fluxes (W/m²); b_x, b_e, coefficients of thermal activity of the semispaces under consideration; $\Gamma(x)$, gamma function; $\operatorname{erfc} x=\frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-t^2)dt$ supplementary probability integral; $\operatorname{ierfc} x=\frac{2}{\sqrt{\pi}}\int_x^\infty \operatorname{erfc} t\,dt$ multiple probability integral; $\binom{n}{m}$, binomial coefficients; $\mathrm{Fo}_x = a_x\tau/r_0^2$, $\mathrm{Fo}_e = a_e\tau/r_0^2$ dimensionless time (Fourier numbers); $\Delta\bar{T}(s)$, $\Delta T(\tau)$, transform and inverse transform of excess temperature at the center of the heated spot; T_0, initial temperature of the system of bodies under consideration; $\bar{\varphi}$ (s), φ (τ), transform and inverse transform of the occurred function (in the text); n, m, summation indices for a double series; r_0, radius of the thin circular heat source.

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